Book reviews

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Ref.: Two Springer University Texts
by Jürgen Jost (MP-Institute for Mathematics in the Sciences), entitled:
I. Postmodern Analysis
A Mathematics Graduate Text. Translated from German by H. Azad. First edition, 1998; circa 350 pp. [ISBN 3-540-63485-1 (pbk)]
II. Riemannian Geometry and Geometric Analysis
An Advanced Monograph. Revised second edition, 1998; circa 450 pp. [ISBN 3-540-63654-4 (pbk)]

With the passing[1] of two renowned figures in the forefront of French Mathematics, Jean Leray (Coll. de France) and André Weil (of the Bourbaki group) – both of whose lives spanned the years 1906 to 1998 – and the recent setting up of the MP-Institute for Mathematics in the Sciences in Liepzig, it perhaps behooves us, as physical scientists with various additional mathematical interests, to look at the recent systematic development of "Analysis" and of topics involving "Geometric Methods on Manifolds". Prof. Jost's graduate text (I) and his more advanced monograph (II) above seem to fulfill a definite need for adequate reference material for the scientist wanting to appreciate the conciseness of functional analysis from a topological viewpoint – one which owes much to the earlier pioneering work of Jean Leray. In contrast, text II gives an extensive background to the role of modern geometric mathematical methods in physics, which are widely used today in quantum-field theory. This reviewer was immediately struck by the clarity, and conciseness of both texts. They also exhibit a high degree of pedagogical structure in the presentation of their subject matter, so that the reader with a purely scientific background will find signposts to the topics of interest to him or her. It is this facility which makes the texts attractive to a wider community, as invaluable guides to assist interdisciplinary studies.

[1] Historical reference (obituary of Jean Leray): I. Ekeland, Nature **397**, Feb. 11th issue (1999) 482.

I. Postmodern Analysis

The approach adopted in text I seems to be generally in the spirit of Leray's lifetime approach to mathematics, i.e., utilising a systematic generalised viewpoint which simultaneously avoids excessive abstraction. The development of functional analysis is given in the topological framework utilising the concepts of Banach space, in the context of convergence of sequences of (continuous) functions, of the Lebesgue integral and of differentiation being taken over a manifold. The first part gives the general formalism defining calculus on Banach space, with examples based on Euclidean space being taken as a subset, in chapter III by way of illustration. Prior to these examples, the concept of metric spaces is introduced in the context of modern topological treatments, utilising the idea of compact sets. The uniqueness of ordinary differential equations is examined in terms of the Banach fixed-point theorem.

Thereafter in the second part of the text (chapters IV–VI), the theory of the Lebesgue integral for Euclidean spaces is developed over a series of subsections, with the most noticeable of these being concerned with convergence theorems, the theorem of Egorov, and the transformation properties of multiple integrals under diffeomorphisms. Thence, one is led naturally into the study of L^p and Sobolev spaces. These topics serve as an introduction to elliptic partial differential equations and hence to the calculus of variations. The discourse in this final chapter is notable for its discussion of physical examples, e.g., by the inclusion of a treatment of the eigenvalue problem associated with the Laplace operator. The book imparts a useful working knowledge of key methods and provides a valuable summary of contemporary analysis, which are especially relevant to several areas of present-day physics.

II. Riemannian Geometry and Geometric Analysis

Second enlarged edition, 1998

This more advanced text provides a synthesis of geometric and analytic methods in the study of Riemannian manifolds and is valuable in giving an overview of modern geometrical methods which have been much utilised in physics of late, but still remain an area which is elusive to many working scientists. This second edition of the text contains new chapters covering the variational aspects of quantum field theory, with a treatment of the Ginsberg–Landau (G–L) functional, as well as a full description of "spin geometry" in the context of the Dirac operators. The presentation is enhanced by the inclusion of subsections (e.g., p. 75, entitled: Perspectives) placing the development, or application, of the material in a wider (or historical) perspective. The foundational summary of manifolds, Riemann metrics, vector bundles and the connection between Lie algebras, Lie groups and "spin structure" sets the scene for a discourse on De Rham cohomology. A subsequent chapter is devoted to the mathematics of *connections*, as in vector bundles, or as a metric, or in terms of the Levi–Civita connection. These discussions serve to introduce the connection between the (so-called) “spin group structures”, the Dirac operator, and the geometry of (minimal) submanifolds. Chapter IV develops geodesic concepts associated with Jacobi fields. This provides an introduction to a useful survey (chapters V–VIII) of curvature and topology, starting from ideas of Morse theory and closed geodesics. Various ideas on symmetric spaces and Kähler manifolds are given before an extensive discourse is developed on harmonic maps.

The final chapter discusses variational problems found in quantum field theory and introduces the Ginsberg–Landau functional and a further form, the Seiberg–Witten functional, associated with Yang–Mills gauge theory for $n = 2$ supersymmetric QCD. The latter functional is notable for having interesting invariant and simplistic forms. The concluding portion of the text demonstrates perhaps most clearly the extent of interdependence between the physical sciences and mathematics. Even for those of us with interests outside of particle physics (or areas involving gauge theories), the text gives a flavour of why geometric methods are of such value in physics, besides introducing the reader to a number of active research areas of modern mathematics. Finally, the introductory material provides valuable concise summaries of a number of topics of wider physical interest that are not easily found in the physics literature. Hence, this concise approachable book represents a valuable reference source, even for the non-specialist reader, or those with essentially scientific interests.